Paper Reference 9MA0/01
Pearson Edexcel
Level 3 GCE

Mathematics
Advanced
Paper 1: Pure Mathematics 1

Wednesday 5 June 2019 – Morning

Time: 2 hours plus your additional time allowance.

YOU MUST HAVE

Mathematical Formulae and Statistical Tables, calculator

ITEMS INCLUDED WITH QUESTION PAPER

Diagram Book

Answer Book

Y58353A

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book or on the separate diagrams – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 14 questions in this Question Paper.

The total mark for this paper is 100

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the Answer Book provided.

1.

$$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a

(Total for Question 1 is 3 marks)

2. Refer to the diagram for Question 2 in the Diagram Book.
It shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians.

(a) Use the diagram to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.
(2 marks)

(continued on the next page)

2. continued.

Given that the root of the equation is α, and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3 marks)

(Total for Question 2 is 5 marks)

3.
$$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\frac{dy}{dx} = \frac{A}{(x+1)^n}$ where

A and n are constants to be found.

(4 marks)

(b) Hence deduce the range of values for x for which $\frac{dy}{dx} < 0$

(1 mark)

(Total for Question 3 is 5 marks)

4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4 marks)

(continued on the next page)

4. continued.

The expansion can be used to find an approximation to $\sqrt{2}$

Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$

- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\dfrac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

(continued on the next page)

4. continued.

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used
(1 mark)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$
(1 mark)

(Total for Question 4 is 6 marks)

5.

$$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3 marks)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3 marks)

(continued on the next page)

5. continued.

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x-2)^2 + 4x - 3 \quad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \quad x \in \mathbb{R}$$

(4 marks)

(Total for Question 5 is 10 marks)

6. (a) Solve, for $-180° \leq \theta \leq 180°$, the equation

$$5 \sin 2\theta = 9 \tan \theta$$

giving your answers, where necessary, to one decimal place.

[Solutions based entirely on graphical or numerical methods are not acceptable.]

(6 marks)

(continued on the next page)

6. continued.

(b) Deduce the smallest positive solution to the equation

$$5\sin(2x - 50^\circ) = 9\tan(x - 25^\circ)$$

(2 marks)

(Total for Question 6 is 8 marks)

7. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t
(4 marks)

(continued on the next page)

7. continued.

The value of car A is monitored over a 10–year period.

Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information. (2 marks)

(continued on the next page)

7. continued.

The following information is available for car **B**

- it has the same value, when new, as car **A**
- its value depreciates more slowly than that of car **A**

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car **B**

(1 mark)

(Total for Question 7 is 7 marks)

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8. **Refer to the diagram for Question 8 in the Diagram Book.**
It shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$

The region R_1 shown shaded in the diagram is bounded by the curve and the negative x–axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$
(4 marks)

(continued on the next page)

8. continued.

The region R_2 also shown shaded in the diagram is bounded by the curve, the positive x–axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4 marks)

(continued on the next page)

Turn over

8. continued.

The roots of the equation

$3b^2 - 20b + 20 = 0$ are $1{\cdot}225$ and $5{\cdot}442$ to 3 decimal places.

The value of b is therefore $1{\cdot}225$ to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root $5{\cdot}442$

(2 marks)

(Total for Question 8 is 10 marks)

9. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b - 1}$$

(3 marks)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2 marks)

(Total for Question 9 is 5 marks)

10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4 marks)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$"

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2 marks)

(Total for Question 10 is 6 marks)

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11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

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Turn over

11. continued.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,

(2 marks)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leq r \leq 20$, is

$$6 \times 1{\cdot}05^{r-4}$$

(1 mark)

(continued on the next page)

11. continued.

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.
(4 marks)

(Total for Question 11 is 7 marks)

12.

$$f(x) = 10e^{-0 \cdot 25x} \sin x, \qquad x \geq 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4 marks)

(continued on the next page)

12. continued.

Refer to the diagram for Question 12(b) in the Diagram Book.

It shows a sketch of part of the curve with equation $y = f(x)$

(b) Sketch the graph of H against t where

$$H(t) = \left| 10e^{-0 \cdot 25t} \sin t \right| \quad t \geq 0$$

showing the long–term behaviour of this curve.

(2 marks)

(continued on the next page)

Turn over

12. continued.

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.
(3 marks)

(continued on the next page)

12. continued.

(d) Explain why this model should not be used to predict the time of each bounce.

(1 mark)

(Total for Question 12 is 10 marks)

13. The curve C with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)}$$

$$x \in \mathbb{R},\ x \neq -3,\ x \neq 2$$

where p and q are constants,

passes through the point $\left(3, \frac{1}{2}\right)$ and

has two vertical asymptotes with

equations $x = 2$ and $x = -3$

(continued on the next page)

13. continued.

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3 marks)

(continued on the next page)

13. continued.

Refer to the diagram for Question 13(b) in the Diagram Book.

It shows a sketch of part of the curve C

The region R, shown shaded in the diagram, is bounded by the curve C, the x–axis and the line with equation $x = 3$

(b) Show that the exact value of the area of R is $a \ln 2 + b \ln 3$, where a and b are rational constants to be found.

(8 marks)

(Total for Question 13 is 11 marks)

14. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2 marks)

(continued on the next page)

14. continued.

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i)

(2 marks)

(continued on the next page)

14. continued.

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3 marks)

(Total for Question 14 is 7 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER